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Source: *The Galpin Society Journal*, Vol. 49 (Mar., 1996), pp. 113-142
Published by: Galpin Society
Stable URL: http://www.jstor.org/stable/842396
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ARTHUR H. BENADE & DOUGLAS H. KEEFE

The Physics of a New Clarinet Design

DURING the 1980s, Arthur H. Benade was active in applying his lifetime's knowledge to the design of new woodwind instruments, and in particular to a new clarinet design that he called the NX (or New experimental) clarinet. His untimely death in 1987 prevented the publication until now of the scientific issues underlying Benade's process of design. This report collects together Benade's writings and research on this subject. The physical principles behind the question of what makes a woodwind excellent are described in a companion report (Benade, 1994) along with a review of the historical development of the clarinet. The companion report, the woodwind chapters in the re-publication (1990) of Benade's *Fundamentals of Musical Acoustics*, and the article on the clarinet by Benade and Kouzoupis (1988) serve as introductions to the present work.

Section I presents an overview of the factors that influence clarinet tuning. This section is based upon the detailed lecture notes of Benade (1977) prepared for a course on the acoustical evolution of woodwind instruments. It summarizes Benade's measurements on historical and modern clarinets that served as the basis for his new design ideas. Section II is a description of the NX clarinet design considerations, adapted from notes dictated by Arthur H. Benade to Virginia Benade in the summer of 1987. The personal style and technical content of the original communication have been largely retained. The minor editorial changes that have been made (by Virginia Benade and Keefe) include small changes in wording and a condensation of theoretical material on second-order effects and on the acoustics of conical waveguides. For a more complete treatment of conical waveguides, see Benade (1988). A summary of Benade's measurements of the NX clarinet is presented in Section III. Sections I and III were written by Keefe based upon written lecture notes and transparencies from various presentations by Benade. An appendix, written by Keefe, provides additional information on topics in the main text.[1]

It is painfully obvious that a complete exposition of the NX clarinet has not been given here. This report is limited to a description of the physics that was important to Arthur Benade during the process of designing a new clarinet, a presentation of available experimental data on the NX clarinet compared with other clarinets, and occasional anecdotal references to the instrument's response to the player. This report does not

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address issues related to key mechanisms, nor does it present sufficient information to allow others to replicate its design. It is hoped that the report will be of value in summarizing the interesting physics involved in modifying the design of a woodwind and in documenting particular aspects of the NX clarinet design for future advances by scientists, instrument-makers, and musicians.

I. TUNING A CLARINET

The overall tuning of a clarinet is influenced by both the open and closed tone holes, the cross-sectional area of the main bore of the clarinet, the position of the register hole, and the design of the bell. It is also influenced by the player's embouchure and the reed/mouthpiece design. These relationships are presented in this section.

To meet a clarinet

The playing demonstrations described here for the B♭ soprano clarinet are generally similar to those used for the other members of the clarinet family. Departures for other members are that the low-pitched clarinets give their players less range of adjustments of reed frequency, and that the A clarinet behaves differently because the same reed is used on a slightly lower-pitched instrument that has a slightly lower cutoff frequency. We examine the steps involved in learning how to meet a clarinet.

1) Play the second-register note E_5, whose nominal playing frequency is 587 Hz, and centre up the note. (All pitch names correspond to the written pitch class and octave, not the sounding pitch.)[2] It turns out that this note has a unique setting of the reed resonance frequency. There is only one way the reed resonance f_r can be aligned with the harmonics of this note. If the player attempts to set the reed resonance to feed the third harmonic, the desired frequency (1762 Hz) is too low to reach without severe strain. The fifth harmonic at 2935 Hz is just barely reachable, but it cannot keep the oscillation alive without the register hole. It is energetically favourable for the player to align the reed resonance near the fourth harmonic at 2350 Hz, remembering that even and odd harmonics are of approximately equal strength in the mouthpiece spectrum above the cutoff frequency (1500 Hz).

2) Make sure the embouchure is properly set by ascertaining that the note persists at *mezzo-forte* level with the register hole closed, but restarts at the low-register A_3 if the reed is touched with the tongue. A slight slackening of the embouchure from this proper setting also produces a drop to the A_3 note, as does a slight tightening. A proper alignment of the reed resonance with an upper harmonic supports the E_5 note even though the register hole is closed. If the reed resonance is not aligned with an upper harmonic of the clarion-register note, then the threshold condition for the cognate chalumeau-register note (A_3 in this case) is more easily satisfied and a shift in register occurs.[3]

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3) Play quickly down the scale from a properly centred E_5 to B_4, slackening your embouchure as you go. (Do not try to maintain the intermediate notes with the register hole closed as yet!) Blowing at a *mezzo-forte* to *forte* level, find the centred note that is confirmed by your ability to maintain the B_4 without using the register hole. This note is supported by energy injection at the fundamental (440 Hz) and the fifth harmonic (2200 Hz), the latter corresponding to maintaining the reed resonance at the fifth harmonic as you play down the scale. It is also possible to align the reed resonance to the sixth or seventh harmonic, 2640 or 3080 Hz, respectively. Note that the B_4 note can centre and sing at a higher and at a lower pitch! This is musically important.

4) If you centre the E_5 again, as in step 1, and then tighten (or slack off) enough to repeat the performance at D_5, your reed frequency will be near 2640 Hz, which feeds the fifth harmonic of D_5. Without a change in embouchure, a sudden closing of the holes to finger B_4 again shows it is possible at *forte* playing levels to maintain the B_4 note in the clarion register without opening the register hole, this time by way of the reed resonance aligned at the sixth harmonic of the B_4 fundamental. The playing frequency of this B_4 is a little higher than that obtained in step 3.

5) Pinching hard will weakly centre the B_4 by the alignment of the reed resonance to the seventh harmonic (3080 Hz), but it is usually impossible to maintain the note without opening the register hole.

6) Return to E_5 and reset your embouchure as in step 1. Now play up the scale with slowly tightening embouchure to G_5, whose fundamental sounds at 698 Hz. This procedure maintains the alignment of the reed resonance with the fourth harmonic of the note, ending at 2792 Hz. It is possible in this case to maintain the G_5 with the embouchure hole closed. A more centred and 'clarinetish' sound, characterized by strong odd harmonics, can be obtained by playing a little flat to align the reed resonance with the third harmonic in the vicinity of 2094 Hz. This reed resonance frequency is getting too low for real comfort.

7) Notice that when the reed resonance is set at about 2792 Hz to feed the fourth harmonic of G_5, a setting results that will feed the third harmonic of the C_5 above, or the fifth harmonic of the $D\sharp_5$ below (in the mouthpiece pressure spectrum, the odd and even harmonics of the clarinet above the 1500 Hz cutoff frequency are approximately of equal amplitude). It is worth some careful thought about the implications of this remark (and of step 4 above) for the tuning of a clarinet by its maker and for the intonation behaviour of the instrument in the player's hands.

8) A procedure very similar to all the above shows that B_5 is best fed with its reed resonance aligned to the third harmonic (2640 Hz) of the playing frequency. Recognize that the same embouchure setting will very nearly centre up $F\sharp_5$ via input to its 2640 Hz fourth harmonic or the fifth harmonic of D_5.

It is very instructive to follow through and to observe by playing

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demonstrations the behaviour of other parts of the scale, recalling that the reed resonance frequency of the soprano clarinet can be set in the range of 2000–3000 Hz without strain.

Tuning curves for the clarinet

Consider a standard clarinet fingering in the chalumeau register in which all of the upstream tone holes are closed and all the downstream tone holes are open. The simplest model of a clarinet is that it acts acoustically as a cylindrical tube open at the end. Such a model predicts that the first and second resonant modes of the air column, f_1 and f_2, respectively, lie in a 3:1 ratio. The mode ratio f_2/f_1 on the logarithmic frequency axis is normalized to 0 cents when this harmonic relationship is satisfied.

Such a simplified model mistakenly assumes that the lattice of open holes is acoustically equivalent to a constant length correction over all frequencies. A physical analysis of the lattice properties of a clarinet-like air column shows that the open hole lattice length correction C is nearly constant at low frequencies with a value of

$$c = s\left[\sqrt{1 + 2t_e/s(a/b)^2} - 1\right],$$

where the inter-hole spacing is $2s$, the radius of the main bore is a, the radius of the tone hole is b, and the effective length of the open tone hole is t_e. The open tone-hole lattice cutoff frequency f_c is given in the long-wavelength limit by

$$f_c = \frac{c}{2\pi}\,\frac{b/a}{\sqrt{2st_e}}$$

At higher frequencies near f_c = 1500 Hz, the open-hole lattice length correction increases (Benade, 1990, p.450). This increase in the length correction corresponds to a decrease in the second-mode frequency f_2, and thus a decrease in the mode ratio f_2/f_1. Fig.1 shows this mode ratio as a function of the low-register written note name. The mode ratio is

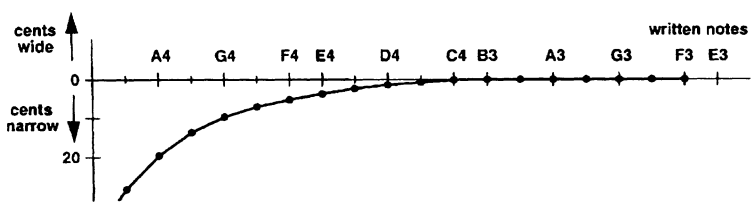

FIG. 1. *Ratio (in cents) for the first- and second-mode resonant frequencies in a clarinet as a function of the low-register written notes, taking into account the perturbation associated with the open holes.*

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narrow for the upper notes in the chalumeau register (the throat notes) because the second-mode frequency is approaching the cutoff frequency. The frequency dependence of the length correction approaches significance when f_2 exceeds one-half the cutoff frequency (750 Hz), that is, for fingerings above written D_4. The tuning error exceeds 20 cents in some of the throat notes.

Now consider a cylindrical pipe provided with a row of closed holes. In the region of these closed holes, the main bore acts acoustically larger and slightly elongated (Benade, 1990, p.448) by a common factor. Specifically, the equivalent area of a closed tone-hole section of length t of the main bore is larger than the geometric area by a factor E given by

$$E = 1 + \frac{D_c}{2},$$

where

$$D_c = (b/a)^2 \frac{t}{2s}.$$

The factor $D_c \approx 0.08$ for typical clarinet designs. The elongation factor is also identical to E. The original cylindrical shape (up to the approximate position of the first open tone hole) is transformed into an air column composed of the original upstream portion including the mouthpiece and barrel joint and a downstream portion with an acoustically enlarged bore. Such a perturbation in the main bore reduces the mode ratio f_2/f_1 (Benade, 1990, pp.473-6). The intuitive explanation is based upon a comparison of cylindrical and conical bores. The first two resonant modes of a cylindrical bore open at the end are in a 3:1 ratio, while those of a complete conical bore open at the large end are in a 2:1 ratio. Consider a perturbation of an original cylindrical tube. Increasing the diameter towards the open end of the tube makes gives it an effective conical taper, and thus reduces the mode ratio. This calculated mode ratio reduction is illustrated in Fig.2 across all fingerings in the chalumeau register. The solid line shows the tuning error for a constant factor D_c, and the dashed line is for a factor D_c that grows at the lower end, as in

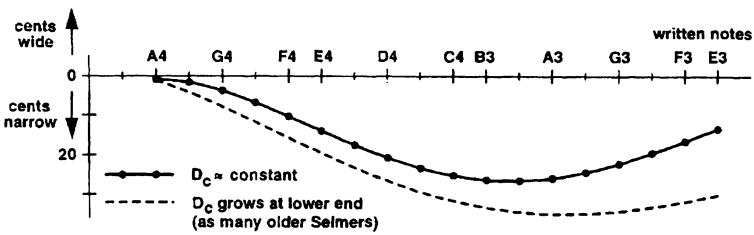

FIG. 2. *Ratio (in cents) for the first- and second-mode resonant frequencies in a clarinet as a function of the low-register written notes, taking into account the perturbation associated with the closed holes.*

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many older Selmer clarinets. The instrument-designer must progressively move tone holes upstream for lower written notes to compensate the elongation effects. That is, the frequency f_1 is affected by this elongation, and the mode ratio f_2/f_1 is controlled by the enlargement. These open and closed tone-hole tuning perturbations were calculated by Benade (mostly before 1968) and compared with measurements on many instruments.

The reed-and-mouthpiece equivalent volume of a clarinet is not quite constant over its frequency range (Benade, 1990, p.472). Viewed as a perturbation of the main bore cross section, the mouthpiece section of the clarinet functions as an acoustical enlargement of the clarinet cross section at its immediate upstream end. When translated as before into perturbations in tuning, Fig.3 shows that the mode ratio is greatly reduced for the throat notes.

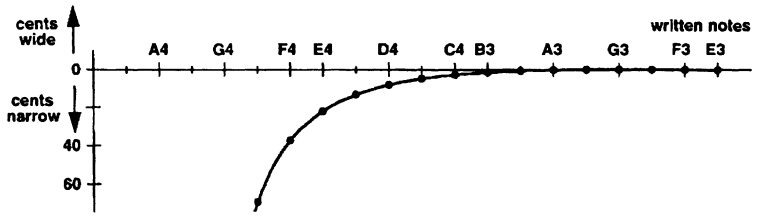

FIG. 3. *Ratio (in cents) for the first- and second-mode resonant frequencies in a clarinet as a function of the low-register written notes, taking into account the perturbation associated with the reed-and-mouthpiece equivalent volume.*

A final influence on the mode ratio is contributed by the frequency dependence of the viscothermal damping mechanisms within the main bore of the clarinet. The phase velocity is reduced more appreciably at low frequencies than at high frequencies, and thus the mode ratio f_2/f_1 is widened. A reasonably good approximation for the clarinet is that there is a widening of approximately 10 cents across all written notes in the chalumeau register.

Taking into account the perturbations on tuning arising from all these factors, the composite tuning curve for a typical clarinet is illustrated in Fig.4. When the mode ratio is within approximately 10 cents of the ideal 3:1 ratio, then there is sufficiently robust cooperation between the reed and the lowest two air column resonances to produce a musically satisfying note. This clarinet would have abominable throat notes, a poor G_4, a tolerable F_4, and superb notes to the bottom of the scale beginning from D_4.

It is now time to ream the upper bore to reduce the discontinuity in the main-bore diameter contributed by the closed holes and to make any other gains in tuning that might be possible. Three simple examples in Fig.5 produce cross-sectional area perturbations that translate into mode

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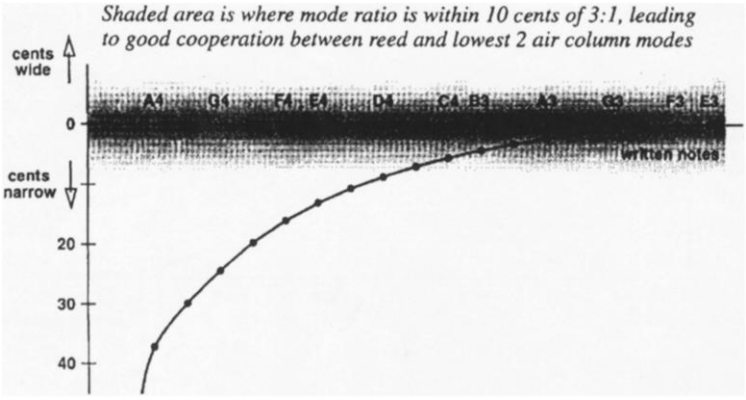

FIG. 4. *Ratio (in cents) for the first- and second-mode resonant frequencies in a clarinet as a function of the low-register written notes, taking into account the perturbations associated with the open and closed holes, the reed-and-mouthpiece equivalent volume, and viscothermal effects.*

ratio perturbations. In each case, reaming the upper end widens the mode ratios, tending to counteract the narrow mode ratios shown in Fig.4. Reaming a cylindrical enlargement down to the position of the G♯ hole, as in Type-A reaming, makes up for the closed hole perturbation, but does not correct the overall tuning errors. Reaming a cylindrical enlargement only half-way down, as in Type-B reaming, does a better job of correcting the errors up to $F\sharp_4$ or so, but is still inadequate. Using a conical reamer, as in Type C, down to the position of the G♯ hole might also be beneficial in contributing a reasonably constant widening of the mode ratio. The perturbations for cylindrical and conical reaming are additive.

Thus far, all the emphasis has been on achieving harmonicity for the low-register notes. Such harmonicity is desirable to maximize cooperations between modes in sustaining the clarinet tone. It is also essential to consider the clarion (second) register, for which the influence of the single register hole of the clarinet must be considered. The clarion-register notes are at playing frequencies in the vicinity of the second resonant mode of the air column.

On a normal clarinet, back at least to the year 1800, the register hole is placed close to its ideal position for producing the A_3-to-E_5 transition into the clarion register. Mode 2 of the air column is then pulled sharp in the manner indicated in the dashed line of Fig.6 for all other notes in the clarion register of a typical clarinet.

It seems reasonable that a designer might arrange a clarinet by watching only the tuning of 12ths played at *pianissimo* levels; this dynamic level would be chosen to eliminate cooperation between resonance peaks that would also influence tuning. The strategy would be to design a

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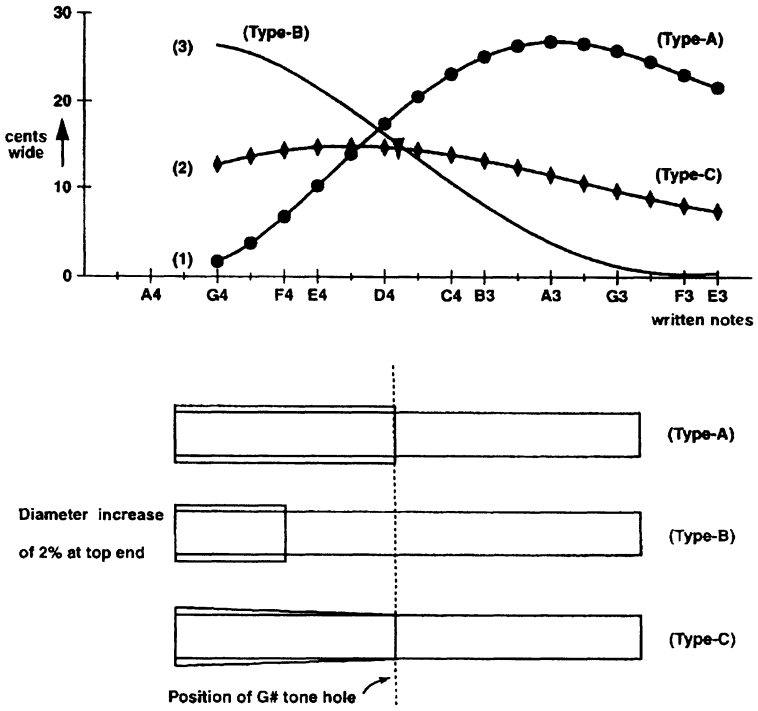

FIG. 5. *Ratio (in cents) for the first- and second-mode resonant frequencies in a clarinet as a function of the low-register written notes, taking into account the perturbation associated with reaming the upper joint of the clarinet. Three types of reaming are illustrated along with their calculated tuning perturbations.*

3-to-1 frequency ratio between the undisturbed first-mode peak and the shifted second-mode peak, that is, shifted after the register hole is opened to eliminate the regime of oscillation near the first-mode frequency. This strategy is almost exactly what happened in the 1940s after the invention of precise electronic tuning meters, before people realized that such a device is a tool and not a master, and that both tone and response are important.

In the spirit of the other mode-ratio shrinking diagrams, tuning a clarinet to the above specifications requires that the perturbations associated with closed holes, open holes, reed-and-mouthpiece, viscothermal losses, and finally bore-reaming of the air column give rise collectively to a curve that is the precise inverse to the one in Fig.6. That is, it is 0 cents at E_5 (or A_3 in the chalumeau register) and about 30 cents *narrow* at the extreme ends of the scale.

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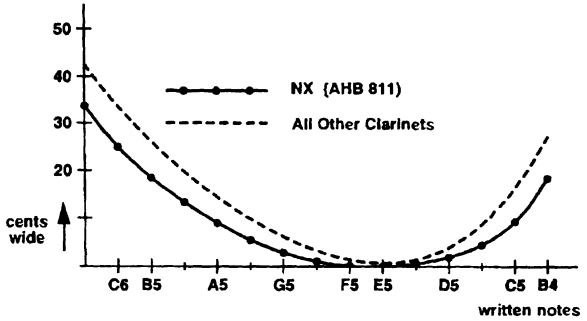

FIG. 6. *Effect of the register hole on tuning of modes 1 and 2 for the NX (solid line) and all other modern clarinets (dashed line). These include the modern Buffet and the aligned Boehm-system clarinets in Fig. 8.*

Comparing such a curve to that in Fig. 4, there is a correspondence of the right sort at the upper end of the air column, but it is an under-correction, according to our well-intentioned designer. How do we take care of getting a droop at the lower end of the horn? This is arranged by simply enlarging the bore near the lower end; the bell design contributes to the solution and is discussed later. If we are clever in our reaming by starting somewhere under the right-hand third finger, we can get a droop of the right sort. One usually ends up in practice with a droop of 35-40 cents at the low E_3 frequency ratio.

This design represents an overall air column arranged to play a perfect 12th with a fixed embouchure at a *pianissimo* dynamic level; this is the typical modern design. In practice, this gives curious tuning of the scale in one register if it is smooth in the other. It also has *lousy* response in the low register except around D_4 to A_3. Moreover, it also runs flat on a *crescendo* in the low register, except around A_3. Now we are in a real mess! At *forte* levels, the low register runs away from the pitch for one reason. If one tries to get good tone in the clarion register by centring up the reed resonances, it turns out that both ends of the scale run a trifle flat (around 10-15 cents). Some makers put in this bore profile to make the 12ths in tune at some *mezzo-forte* playing levels, which makes better practical sense than the scientifically easier *pianissimo*-level tuning discussed earlier. But, one still has all sorts of quirks in the tuning, note by note.

A better way

A better way to arrange things was fairly common in clarinets made before 1910 and is often found in German clarinets today. These ideas are intimately connected to the NX design.

1) Ream the upper bore such that the *pianissimo*, fixed-embouchure 12ths are about 10-15 cents wide. The exact adjustment is made to put

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C_6 in tune when the register hole is open and the embouchure is slackened to align the reed resonance at the second harmonic (1864 Hz). This gives a little assistance to the 6th harmonic of F_4 in the corresponding chalumeau register, if desired. Its tuning should be centred at the *mezzo-forte* playing level.

2) Tinker the lower bore so that only a little slackening in the embouchure will let the player bring B_4 into tune, putting the reed resonance at 2200 Hz where it can feed the 5th harmonic of the note and so, being an odd harmonic, giving it a more clarinetish sound than one might otherwise get from the bell note.

3) In between these limits, things are straightforward.

4) Note that the designer must arrange things so the player can in practice pull up 12-15 cents or down 12-15 cents without loss of security, so as to be able to fit in all tonal contexts with appropriate tempered intervals.

Digression on tempered tunings

There are very important relationships between intervals in equal temperament and the musical exact intervals, which are beat-free, that are preferred when time and circumstances permit. When they are summarized over a number of tempered and beat-free tunings, these beat-free intervals often differ by approximately 10 cents above or below the corresponding tempered intervals. We can thus understand why it is that many musicians form the habit of 'thinking' a note sharp or flat relative to equal temperament. While playing, such a musician starts with the written note, recognizes from the context whether it should be played in the middle, or above or below the equal-tempered pitch, and finally zeroes in to a more exact setting if there is time. If there is not time or if the listening conditions are not suitable, the player nevertheless has got within a very few cents of the musically desirable setting, and by very simple means. Most players are unaware of the arithmetic of what they are doing, but because of long habit they will almost always go up or down in sounding frequency by about 10 cents if asked to play a little sharper or a little flatter. Because of the need to move up or down a little from equal temperament and because instruments do not 'sing' well if their pitches are pulled too far above or below their own inherent tuning, the players of wind instruments take pains to find instruments that are built to play most naturally in accordance with the needs of the middle, upper, and lower clusters of tunings that are required.

The skilled musician quickly finds the playing pitch at which the instrument 'sings' best. The pitches that are associated with best tone, promptest response, steadiest sounding frequency, and widest dynamic range do not always exactly coincide with equal temperament or with its special-interval relatives. The good-response cues are generally so strong on a good instrument that the player can repeat the sounding frequencies

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far more accurately (even from day to day) when asked to produce the best tone, than when asked to concentrate attention on pitch alone. This fact is of enormous utility in guiding research and controlling the adjustments which improve their overall behaviour.

II. DESIGN CONSIDERATIONS FOR THE NX CLARINET

In the mid-fifties I began to spend considerable time on my interests in the acoustical workings of the woodwinds. [The *I* here refers to Benade, who wrote all of Sec. II.] My first paper on the subject appeared in 1959. By about 1978 or 1979, I had learned enough about the playing properties of a wide variety of clarinets (all the way back to the five-key Mozart-era ones) that I could say I was familiar with them. I was aware of two things: that I didn't have the whole picture and that I already knew a lot. And I had noticed that many of the clarinets made before about 1820 had, from the player's point of view, some rather remarkable virtues – but virtues that only revealed themselves if the player was willing and able to use his 'chops'. Acquaintance with these early-nineteenth-century instruments made it quite clear to me that their design aided the player, in a relatively easy way, to perform the tremendous leaps and bounds common in the lighter-weight music of the time.

I had been successful in designing a conical, Boehm-fingered flute that has a baroque sound and playing characteristics, and I became more and more attracted to the idea of making a clarinet that would retain the virtues of the old instruments while sidestepping some of their problems. At the beginning, I paid little attention to any difficulty arising from the choice of fingering system, because it had always been easy for me to learn another system.

The register-hole system

In the years that followed, a qualitative instrument-maker's and player's design evolved in my head, often during long automobile trips. I knew enough about clarinets acoustically and musically to stay out of physics trouble and to lay out something that would be at least partially playable.

I decided the first item of significant difference in my design would be to return to the old, small-diameter register hole (2-mm bore through a 12-mm-thick wall). The virtues of this change would be apparent to all, leaving me with only the challenge of coping with the problems brought on by doing this. To deal with these problems, one has merely to ordain a separation between the functioning of the register key and the throat $A\sharp_4$ key (something explained in nearly all books on the clarinet).

At the time of the earlier parts of my imaginings, I was quite aware of the functioning of the reed resonance (Benade, 1990, pp.436, 439, 462; Thompson and Benade, 1977). I had noticed something while playing the clarinet: if one pays attention to tone colour alone, the setting of the

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embouchure tension that gives the best reed-resonance effects at the upper end of the clarion register tends to pull the playing pitch down about 15 cents. This happens to be just enough to pull the scale into very nice tune when the smaller register hole is used. In the same situation, with the clarinetist playing for tone colour alone, the normal, large register hole pulls the playing pitch up some 25 to 30 cents. This means that if nothing is done to the bore, a musician who wants to play in tune will have to use lip tension to pull the pitch down. However, achieving this pitch change by lip tension gives a note that is in tune but which is some 10 to 12 cents below what gives the best tone at that lip tension setting. An analogous phenomenon obtains at the low end of the clarion register scale. It is therefore a happy accident of physics (and a fact empirically discovered by the early makers) that a single, small register hole, as described above, will permit the simultaneous achievement of a good scale, including plausible twelfths, and of good tone and response over the entire clarion register (as explained in Section I). And the response of the clarion register is important because it controls the approximate alignment of one reed resonance or another with each note in the lower register.

The fact that one can play off the tone achieved by a properly chosen register hole against the controlled sharpening effect (from changes in register-hole size) in the second register means, first of all, that only one register hole is needed, instead of the two or more that add greatly to the complexity of today's bass clarinet. For players unwilling to use their chops, present standard practice shows that careful instrument design can give good overall tuning, but only at the expense of tone and response at the two ends of the clarion scale. However, when the smaller register hole is used, it becomes possible for any player who is willing to use his chops to improve the tone at both ends of the clarion register to match that of the already-satisfactory tone found in the middle register.

The design of the instrument described above is in fact rather close to Mozart's clarinet, which (because it had few keys) paid for these virtues by having a very bad $A\sharp_4$ along with other rather unsatisfactory throat notes.

Nothing has been said so far about what guides an instrument-maker's choice of a basic shape for an instrument's air column. If one were concerned only with the clarion register, it could be of any shape that was even vaguely cylindrical, and the scale could be brought into tune by the selection of tone-hole positions and sizes. Note that any virtues of tone colour and response in the clarion register can be achieved without great difficulty by the mutual proportioning of bore and tone holes, without destruction of a usable playing scale. This flexibility comes from the fact that the only resonances that need be aligned are the second-mode resonance of the air column and the reed resonance, and these are under the control of the player, who can set them anywhere between 2000 and 3000 Hz.

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The freedom to choose any available air column shape for exploitation in the second register is not available for the low register. In the low register, the regimes of oscillation require the cooperation of the mode-1 and mode-2 resonances, so that the air column shape must be proportioned in conjunction with the volumes and spacings of the closed tone holes that essentially are always present when a note is played.

For proper low-register regeneration (using here the language of physics), mode 2 of the desired air column must have its frequency in exact harmonic (3:1) relation to the frequency of mode 1; it is obvious that such a tube is, in essence, cylindrical, with an excitation mechanism that functions as though the tube is closed at the top. We have just described the physicist's prototypical clarinet, which, of course, overblows to the musical interval of the twelfth. The main effect of a row of closed tone holes is to make the air column cross section effectively larger in the region of these closed tone holes; the instrument-maker compensates for this by enlarging the diameter of the bore itself in the regions above and below the main part of the instrument body where the holes are arranged.

Non-linear effects

The second (and again quite visibly physics-related) set of phenomena that influenced the evolution of the design of this new/old clarinet was the awareness that non-linear effects in the air column and its accompanying tone-hole system were continually showing themselves in the domain of practical work with instruments. However, they had been stubbornly invisible in laboratory measurements, in large measure because none of the drive mechanisms available with the various impedance heads then current (see Benade and Ibisi, 1987) were able to attain signal levels that would reach into the mathematical domain of the second-order wave equation. And there was a great deal of experimental work still to be done using playing experiments that made use of the domain of the first-order theory.

It was generally sufficient to keep an eye on the appearance of non-linear phenomena in the course of any playing experiments, and many such experiments were carried out at a high level of musical dynamics. It is not difficult to attain levels of 140 dB SPL in an ordinary clarinet mouthpiece on a simple 250-mm piece of clarinet tubing when it is played with a medium-soft reed at maximum possible blowing pressure (Richards and Benade, 1982). More SPL is achieved in a well-made trumpet.

In the latter part of this period of work in Cleveland, Douglas Keefe (1980) had occasion to look scientifically into the possible implications of the non-linear acoustics for the behaviour of musical instrument air columns. The second-order acoustic wave equation includes source terms that take their excitation from the squared-pressure aspects of the

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linearized equation. One of these terms varies as the square of the particle velocity and is closely related to the quadratic Bernoulli non-linearity that attains a simple form in problems of potential flow. This term may usefully be referred to as the complicated-flow term, which becomes appreciable only in waveguides having boundaries of complicated shape. Perhaps no one but a woodwind acoustician would even think of allowing himself to be entangled with an air column system having as many corners and irregularities as those belonging to the tone-hole system of most woodwinds!

The linearized conservation-of-momentum equation states that the time-derivative of the particle velocity is proportional to the space derivative dp/dx of pressure, and this governs the scale of the complicated flow term. It takes only a moment to recognize that the general size of the space derivative dp/dx is determined, in a tone-hole lattice, by the ratio p/b,where b is the radius of a tone hole. We can also recognize that the general extent of a flow complexity is limited to a distance that is some fraction of the air column radius, simply because the characteristic length over which the evanescent mode responses extend is of this order at low frequencies. These considerations indicate that whatever complex flow effects may be active in a tone-hole lattice, they are appreciably active only at three places: in the immediate neighbourhood of the tone-hole entrance into the bore, at the exit of the tone hole into the room, and, quite significantly, in the region under a pad that has been opened, unless it is lifted quite high. Note that, within the tone hole, whatever disturbance is set up at its inner end will smooth itself out on its way to the outside over a distance somewhat less than the hole radius unless the chimney of the tone hole is short. In that case, the outward extension of the inner disturbance can overlap and interact with the inward extension of the external disturbance, thereby 'complicating the complexity' and drastically increasing the magnitude of any non-linearity.

At this time both Keefe and I, for our own reasons, began keeping track of the first-order viscous losses taking place in the air column. It is a commonplace in the musical acoustics laboratory that many physical phenomena do not display themselves in a direct physical measurement until enough physically guided 'player's experiments' have been carried out to map out the territory and give an idea how to make a measurement. This points up the fact that the domain of a player's sensitivity and the manner in which he or she detects a phenomenon can be widely different from those belonging to the physicist.

These considerations led us to devise a pair of tone-hole lattices. Everything in classical transmission-line theory depends on the impedances of the holes, on the spacing between them, and, to a limited but familiar extent, on the small damping effects within the tone-hole lattice. Because of these considerations, our two lattices were designed to have the spacing of the holes (which are uniform) of one of the lattices

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identical with the spacing of that of the other. Another design requirement was that the imaginary part of the impedance of the tone holes should be the same in both lattices. One lattice was provided with small holes that had short chimneys, while the other was given large holes and tall chimneys. The newly developed theory of woodwind instrument tone holes (Nederveen, 1969; Keefe, 1982a; 1982b) made it easy for us to calculate, in advance, reasonably accurate *a priori* values for the effective length of the two sizes of the tone holes so that the two lattices would be alike (according to first-order theory) and the machine shop would be able to provide us with these two first-order related objects. All this ensured that the tone holes would not need the small trimmings and paintings that might otherwise make the results we were seeking ambiguous. One would clearly expect the first of these lattices to have large complexity effects, while in the other such effects would be small.

When these tone-hole lattices were attached in turn to a piece of clarinet-like tubing as a termination and the input impedance was measured with an ordinary low-level impedance head, the response curves proved to be almost precisely alike. This implied that no differences should be observed between the playing behaviour of these two clarinet-like air columns at sufficiently low dynamic levels such that the linear impedance description is accurate. On the other hand, we hoped that with stronger blowing we might detect at least some evidence of the complexity phenomena.

Keefe, who was at that time a strong saxophone player and familiar with the clarinet, found the tall-chimney version of our 'clarinet' responsive and easy to play and possessed of a very pleasant tone. On the other hand, several trials were needed to get the short-chimney version to sound at all. Benade's own trial of both instrument versions confirmed those observations. The tall-chimney one spoke promptly and easily, while Benade's 'universal embouchure' managed to get the short-chimney clarinet to speak only reluctantly and with a harsh, stuffy, squeaky tone. Very strong acoustical streaming issued from the tone holes of the short-chimney instrument.[4]

This streaming phenomenon had been encountered several years earlier in connection with bassoon experiments sparked by Bill Waterhouse (Benade to Waterhouse, 1972). In these experiments the tenor joint of a bassoon was provided with a replacement set of tone holes in which the top tone hole (F_3) had an impedance that matched that of the normal hole, but it had been drilled through the back side of the joint, where the wood was much thinner. The rest of the new sequence of holes had gradually expanding diameters and spacings that provided a tone-hole lattice of textbook uniformity and an accurate scale, and their cutoff frequency was the 450 Hz that is characteristic of this part of the bassoon scale. The intention was to see whether the

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impedance curves of the air columns produced by closing the uniform set of holes would be closely similar to the curves measured when the set of normal holes are closed successively. It was known from indirect evidence that the ordinary holes, though they are drilled at all sorts of angles and in various sizes, appear to act in the lattice in a very regular way, so a direct check had seemed worthwhile.

As a matter of fact, the two sets of impedance curves, again measured at low level, were almost identical all the way out to well above cutoff. This was of course interesting and happy news, because it helped justify the use of formal mathematical physics for a slowly varying lattice on a geometrically quite irregular physical structure.

When an attempt was made to play the bassoon using the new and uniformly tapering set of tone holes, the sound proved to be extremely stuffy and the instrument was marginally able to play only when blown very hard. For what it's worth, there did not seem to be any disruption of the ordinary oscillatory feedback mechanisms of the instrument, in sharp contrast to what we found in the case of our new pair of clarinets. On the other hand, there was evidence of very heavy damping, so that the instrument felt like one whose resonance peak heights had been lowered by about one-third. Moreover, there was very strong streaming from the first few of the modified open holes on the bassoon, exactly as in the case of the short-chimneyed 'clarinet'. The streaming was no real surprise and, since it was a side issue, it was passed off merely with a comment that streaming was present, and the work continued on other things.

Clearly, our 'player's experiment' on the pair of clarinet lattices demonstrated the importance of the non-linear acoustical field. This was an experiment in which it was known from earlier work that a player could easily detect a one per cent change in either the damping (first order) or in the condition of the feedback loop. What we found was a much more prominent effect than simply a perturbation in tone colour insofar as the stability of the clarinet's oscillation was disrupted.

From the point of view of an instrument-maker wishing to minimize complicated-flow disruptions in the oscillations of his instruments, our experiment has the following major implications. First, if one is to minimize the disruptive interactions taking place at the inner and outer ends of the tone hole, one should keep the length-to-diameter ratio, $t/2b$, reasonably large.[5] The use of open pads over tone holes should be minimized to reduce the additional complexity associated with flow out from under a pad. (As pads began to develop, there was much complaint among musicians that the use of pads 'spoiled the tone'.) Towards the upper end of the instrument where the tone holes get progressively closer together, one should keep the inter-hole distance as large as possible. At the top end, the requirement of keeping the tone-hole lattice cutoff frequency constant makes it very easy to maintain the $t/2b$ ratio at a satisfactory value. In the lower part of the instrument, on the other hand,

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this lattice cutoff frequency requirement leads to ever-larger holes that are often drilled through ever-thinning walls, so that the $t/2b$ ratio may become progressively less favourable.

There is one more point well worth noting that can cause considerable difficulty at the upper end of a modern instrument, whereas it has no effect on the older ones. A slavish following of the precepts of Theobald Boehm gives a situation of 'full venting' in which, for any note, all the adjacent tone holes just below the playing hole are open. Up high on the instrument, the semitone scale sequence places these tone holes close together, so that the complexity belonging to the inner ends of adjacent holes can interact strongly. The outer-end disturbances can similarly interact.

Conical terminations and flaring bells

The third point at which the history of woodwinds touches the acoustics of air columns is the bell. To first order, the bell of a clarinet can be approximated by a suitably chosen cone. Benade (1988) shows how the acoustical behaviour of a straight-sided conical bore segment is equivalent at long wavelengths to the parallel combination of a cylindrical bore and a so-called conicity impedance that acts as a pure inertance.

All this being true, to attach a short length of cone, L_c, to the bottom end of a tone-hole lattice where the cone entry and the main bore radii are both a is to construct a system whose upper part consists of a pipe provided with a row of holes ending at the junction with another 'pseudo-hole' whose proportions are such as to give it an impedance equal to the cone's entry-way impedance. Below this, the pipe effectively continues on its cylindrical course to the end of the replacement transmission line, denoted by the cone length. In other words, if the cone length has been chosen so that it is equal to the lattice inter-hole spacing ($2s$), an observer (or reed system) 'looking down' the tone-hole lattice will think that the lattice has been extended by one unit, the impedance of the bottom hole being determined by the properties of the cone. The simplest approach to fixing a bad bell design is to add another open tone hole before the bell, that is, to literally extend the lattice by one unit, and this has been adopted in many instruments. The drawback is that the instrument is lengthened, and its carrying power is reduced by the increase in viscothermal damping. It also becomes more difficult for the performer to hold. If the taper of the cone is now correctly chosen so that the conicity inertance matches the inertance of the upstream tone hole then the pseudo-hole will appear to the reed to be exactly like the real holes of the lattice.

This means that an abrupt change from cylindrical bore to conical bell forms a near-perfect extension of the tone-hole lattice. Since it is the tone-hole lattice that all-but-completely determines the musical value of

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a woodwind instrument, this means a finite-length instrument can have one more good note in its scale than can an instrument with a smoothly flaring bell of the sort that is familiar today.

In the early Beethoven and pre-Beethoven clarinets, one frequently finds truly conical bells, and it was only later that intellectual efforts at 'rationalization' began to modify the shape toward the more trumpet-like form. It is a familiar fact that in the modern clarinet the notes E_3 and B_4 have a tendency to blare if players do not control their embouchure (these are notes that do not normally bray or honk on the conical-bell instruments). This blaring is especially obvious if one attempts to slur abruptly to these notes from ones that are a considerable distance from them in pitch. The major explanation of this is that these 'blare' notes are produced using the entire air column, and this air column is terminated by a trumpet-like bell whose radiation and reflection properties are quite different from those characteristic of the extended tone-hole lattice that forms the prototype for a woodwind air column. It is a well-known but somewhat surprising fact that to have even one open hole at the bottom of a woodwind instrument will serve in large measure to isolate the effects of a flaring bell. This means that adding a pseudo-hole by use of a conical bell can remove the adverse bell effects on even the bottom note of an instrument's scale. Similar bell-related phenomena occur on the other woodwinds, but the effects are more subtle and their explanations are not germane here.

III. EXPERIMENTS ON THE NX CLARINET

One method of assessing the acoustical quality of an instrument is based upon experiments under playing conditions. The tunings of the chalumeau and clarion registers, based upon modes 1 and 2 of the air column response, respectively, are gauged by measuring their playing frequencies. Such tunings were measured by Benade for a number of clarinets, including the NX (NX 811), a modern so-called Boehm-system clarinet whose air column has been modified to improve its mode frequency alignment (denoted aligned Boehm-system and marked AHB B-20659), and a Buffet clarinet (serial no.219164) typical of modern instruments.

Fig.7 illustrates the frequency ratio (measured in cents) between mode-2 relative to mode-1 on three clarinets for chalumeau-register notes. The mode ratio tends to be narrow (that is, less than 0 cents) for all clarinets, particularly in the upper throat notes. The Buffet clarinet tuning is narrowed by 10-20 cents in the mid-range of the chalumeau register, and this mistuning worsens at the bell end and even more in the throat notes. For example, its mode ratio is more than 50 cents flat at A_4. The aligned Boehm-system clarinet (AHB B-20659) is an instrument that was extensively re-worked by Benade around 1970, and its mode

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ratio lies between the Buffet and the NX. The NX was designed in the early 1980s, and measurements were taken in 1983.

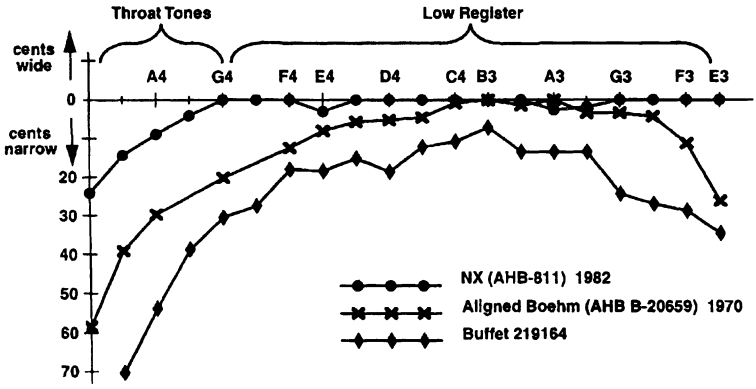

FIG. 7. *Alignment of playing frequencies of modes 1 and 2. Comparison of NX clarinet with a modern Buffet and the aligned Boehm-system clarinet.*

As previously discussed, the deviation of the mode ratio from ideal tuning is dominated by open and closed tone-hole effects and the influence of the clarinet and reed mouthpiece. Indeed, the measured response of the Buffet clarinet is similar in the upper notes to the predicted perturbation in Fig.4, although the Buffet clarinet has much more narrow intervals. The measured response of the Buffet clarinet has an additional droop in the lower notes that is not predicted by Fig.4. This difference is because the Buffet has been tuned according to the typical modern design, namely, such that the instrument plays in-tune 12ths at a *pianissimo* level for all fingerings; under the conditions that the player opens the register hole for the clarion notes and makes no change in embouchure. In part, the NX and the modified Boehm clarinets reduce this narrowing of mode 2 relative to mode 1 by an enlargement in the upper bore, but the influence of the register hole must also be considered.

Opening the register hole to produce the clarion-register note perturbs the frequency of mode 2 and thus modifies the tuning ratio. Slackening the embouchure at both ends of the scale can compensate for this widening effect. This is seen in Fig.6 in that the widening is less for the NX clarinet than for the other clarinets, including both the Buffet and the aligned Boehm-system. This is the essential trade-off between pitch and tonal quality. The NX clarinet design has a different register-hole design and it is expected that the player slackens the embouchure for correct tuning. This allows excellent tonal quality at the extreme ends of the clarion register. Other modern clarinets assume that the player keeps

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a fixed embouchure and sacrifices the tonal response by de-tuning modes 1 and 2.

As a complement to playing experiments, a measurement of the air column input impedance gives accurate information concerning the air column in the absence of the excitation mechanism. Benade measured the input impedance magnitude of the NX and a Buffet clarinet for fingerings across the entire range of the chalumeau and clarion registers (see Figs 8 and 9). He discussed these data in detail with Keefe in 1983-84. These measurements provide detailed information on the uniformity of tonal response across registers and allow detailed examination of the low notes dominated by the bell acoustics, the throat notes, and both extremes of the clarion register. A qualitative overview shows that the impedance plots for the NX clarinet are more uniform and more smoothly varying than those for the Buffet clarinet. In other words, acoustical regularity is a virtue. The chalumeau register should have impedance peaks with well-defined frequency relationships up to the cutoff frequency of approximately 1500 Hz. When performing different notes, acoustical regularity also means that the amplitude of the first impedance peak should be smoothly varying across different notes, and similarly for the second and higher-order impedance peaks.

The impedance for the range of six pitches from C_5 to G_4 includes the throat notes and the transition from the clarion down to the chalumeau register (Fig.8, top). The corresponding six impedance curves are much more irregular for the Buffet clarinet than for the NX. The peak heights of the lowest group of impedance peaks vary more for the Buffet, and there is an abrupt transition in the next highest group of impedance peaks. Specifically, the peak amplitudes for the two clarion-note fingerings (C_5 and B_5) are much lower than those for the four throat-note fingerings. In contrast, the NX clarinet peak heights in the lowest group are uniform and larger than those for the Buffet clarinet (which means the NX air column impedance peaks have higher quality factors and less damping), and the transition in the second group of peaks is smooth between the clarion-note fingerings and the throat-note fingerings.

These trends continue in the range of eight pitches from G_4 to C_4 (Fig.8, bottom). The Buffet clarinet impedance plots are more uniform in this range than in the throat-note fingerings, as would be expected, but the quality factors are lower than those of the NX clarinet. In the frequency range of 1200-2000 Hz that includes the transition below and above cutoff, the NX peaks roll off more smoothly than those for the Buffet. The conclusions are similar for the range of six pitches from C_4 to G_3 (Fig.9, top). In particular, the impedance magnitude response of the Buffet clarinet is more irregular above cutoff (1500-2000 Hz), which is a signal that the term cutoff frequency is a bit of a misnomer. The reason is that reflections from the bell create a ripple at high frequencies with peaks at isolated frequencies that bear no simple relationship to the

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low-frequency peak frequencies. In contrast, the conical bell on the NX clarinet is designed to mimic the continuation of an open tone-hole lattice, and thus the cutoff frequency predicted from the simple physical models is found in practice in the NX clarinet.

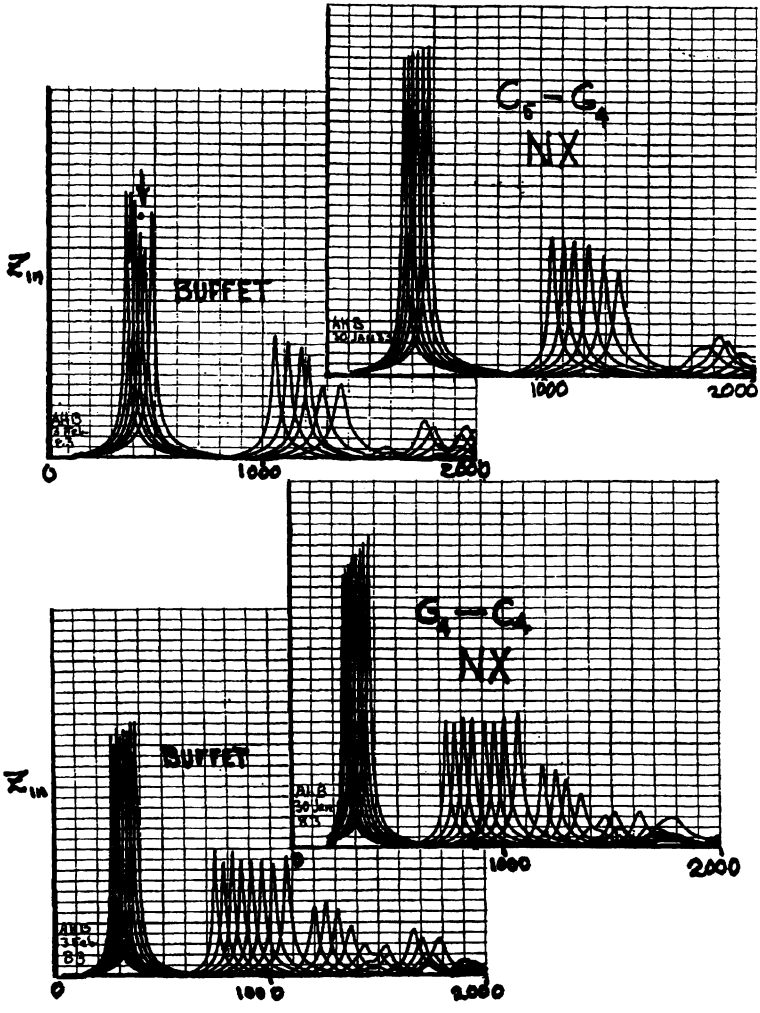

FIG. 8. *Measured input impedance for NX and modern Buffet clarinets versus frequency: the throat notes (C_5-G_4); the upper notes of the scale (written G_4-C_4).*

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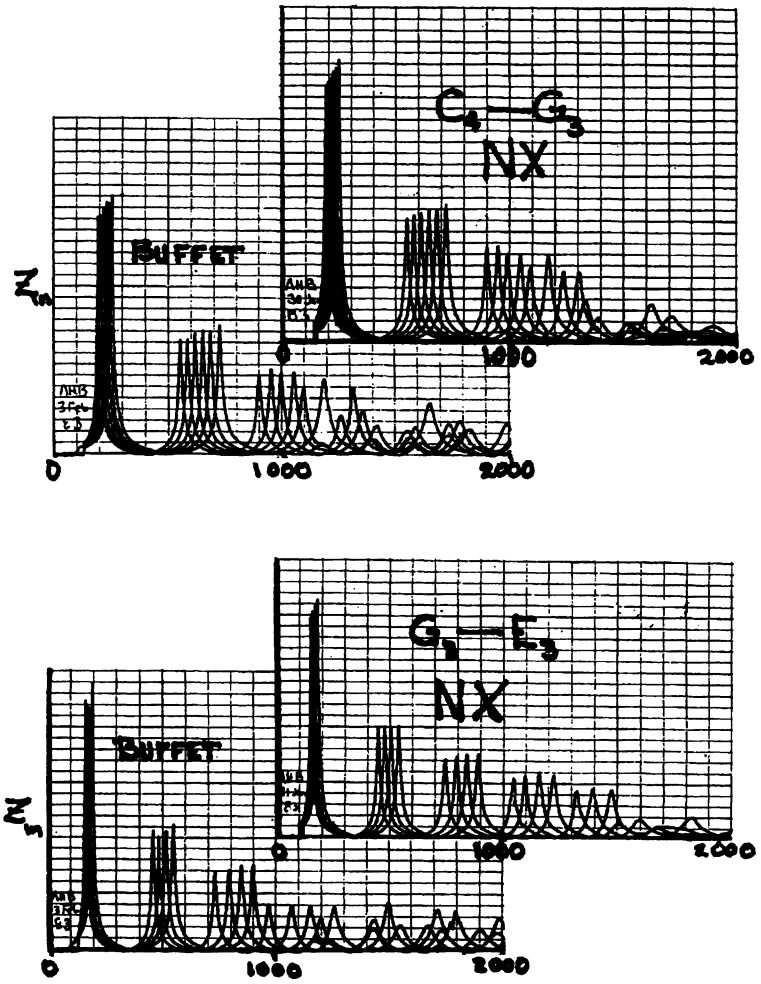

FIG. 9. *Measured input impedance for NX and modern Buffet clarinets versus frequency: the lower notes of the scale (C_4-G_3); and the extreme lower notes of the scale, strongly influenced by bell effects (G_3-E_3).*

In the lowest range of four pitches from G_3 to E_3, these bell effects become more pronounced (Fig.9, bottom). This is a playing range where the tone quality of modern clarinets (including the Buffet) changes rather abruptly, and it is primarily due to the failure of the bell to provide a well-defined cutoff frequency. This is evident in the Buffet clarinet

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measurements, in which the peak amplitudes in the range of 950-2000 Hz are nearly uniform in height with roughly uniform frequency spacing between peaks. In contrast, the NX clarinet impedance magnitude is characterized by four groups of impedance peaks up to a cutoff frequency of 1500 Hz. There is no abrupt change in tone quality in the lowest pitches of the NX clarinet.

Having found that acoustical regularity is a virtue and that it can be explicitly incorporated into a woodwind instrument design, it remains to explore the ramifications of that delightful warning not to complicate the complexities. The requirement to use tone holes introduces complexity into the design of any woodwind instrument. Two forms of complexity have been discussed, one associated with the design of the single tone hole and the other associated with the potential interactions between closely spaced tone holes. While the designer cannot avoid the complexities, it is possible to avoid complicating the complexity. This means several things. Begin with the so-called aligned Boehm-system clarinet, which came into Benade's hands as a typical modern clarinet whose ancestry is irrelevant.

Although detailed descriptions of the modifications to this instrument are unavailable, it is possible to surmise how it was modified. Any of Benade's modifications of woodwinds included careful rounding of the corners of the joints between sections of the air column and the corners of the tone holes. Some of the reasons for this are discussed in Benade (1990). The presence of any discontinuity in the geometry of the internal air column leads to additional non-linear acoustic losses that are undesirable. Benade concluded that the geometry of each tone hole should remain as simple as possible. For example, the widespread open-hole systems on many modern Boehm flutes is undesirable because when the pads are closed, there remains an additional complication in the flow pattern near the hole, and thus additional losses. With respect to the clarinet, he concluded that the tone-hole chimney height should not be too small, again to avoid additional non-linear losses. The barrel joint of the aligned Boehm-system clarinet was reamed in accordance with the mode-perturbation effects described in Fig.5, and the bell was probably modified by the addition of a strategically placed open hole so that the bell response had a similar cutoff frequency to the open tone-hole lattice (adding an extra open hole was an alternative solution Benade sometimes used in place of re-designing the bell to be conical). While some selective modification of tone-hole dimensions was possible on the aligned Boehm-system clarinet, the choice of name suggests that the tone-hole layout was similar to that of a typical Boehm clarinet.

The additional ways in which the NX design simplifies the acoustic complexity of the tone-hole layout are illustrated in Fig.10. The figure identifies the zig-zag layout of closely spaced tone holes just below the register hole. Two tone holes are closely spaced when their inter-hole

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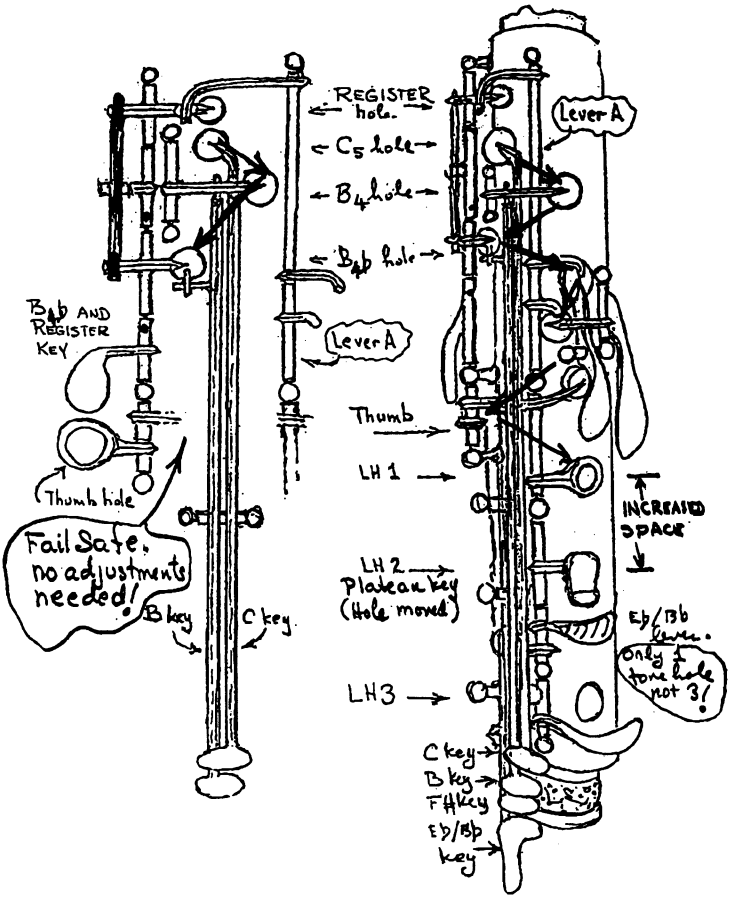

FIG. 10. *Top-end mechanism of A. H. Benade's NX clarinet. The zig-zag arrangement of closely spaced tone holes is shown (at top).*

separation is less than a characteristic length, on the order of the radius of the main bore (7.5 mm for the soprano clarinet). For a single tone hole, there exists a local acoustic field that does not extend much beyond this characteristic length. However, when a second tone hole is brought near to the first tone hole, then the local acoustic fields overlap, and the complexity has been complicated. The mutual influence of these local acoustic fields affects tuning. By zig-zagging the placement of adjacent closely spaced tone holes, the perturbation upon tuning is reduced, and this probably extends to reduced losses as well in the linearized impedance function. This zig-zag design contributes to the relative

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uniformity of the impedance functions of the pitches in the transition between the clarion register to the throat notes (Fig.8, top). In contrast, the Buffet clarinet and other clarinets use a linear arrangement of the tone holes, which tends to maximize the mutual influence of the local acoustic fields.

Even better is when closely spaced tone holes can be removed entirely. An example is shown on the lower right side of Fig.10, in which three tone holes on the typical modern clarinet have been replaced by a single tone hole on the NX clarinet, with modifications in the positions of other nearby tone holes.

The last stage of avoidance of complication comes when the non-linear losses are considered. If there is additional non-linear loss or flow separation downstream from a single tone hole, there is an additional length needed for subsequent re-attachment. The zig-zag design increases the distance between flow separation points as compared to typical designs.

IV. CONCLUSIONS

(by D.H. Keefe)

Arthur Benade's NX clarinet represents a significant achievement across several disciplines. The knowledge of physics was combined with the knowledge of the player, made explicit through systematic playing experiments. In addition, Benade brought together personal insights from making and modifying instruments and an extensive knowledge of the history of clarinet design, thereby re-introducing beneficial aspects of older designs that had been abandoned in the course of evolution. Of course, whether the NX clarinet is an improvement will be decided by future developments in instrument-making and their degree of acceptance by performers. While such a broad combination of skills is indeed extraordinarily rare in one individual, Arthur Benade's accomplishments culminating in the NX clarinet provide an existing proof that interactions between the communities of musical acousticians, performers, and instrument-makers can contribute to the future development of musical instruments.

ACKNOWLEDGMENTS

This report could not have been prepared without the sustained encouragement and assistance of Virginia Benade. Appreciation is also due to Jay Bulen, who assisted in the preparation of the illustrations from the hand-drawn originals.

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APPENDIX

Recent research on sound production in the clarinet

The descriptions of playing demonstrations place considerable emphasis on the role of the reed resonance in adjusting the playing frequency. This appendix reviews the work of Benade (1990) and Thompson (1979) and describes recent perspectives on sound production in the clarinet.

Using a model of the reed as a simple harmonic oscillator, the reed resonance frequency is the square root of the ratio of the reed stiffness to the dynamic mass of the reed. The reed stiffness is relatively well defined under static conditions, whereas the dynamic mass must be defined in terms of the mode shape of the reed deflection and its kinetic energy. These quantities co-vary as the player adjusts the embouchure. When the player tenses the embouchure, the reed tip aperture decreases due to the curved lay of the clarinet mouthpiece, and the reed stiffness increases due to the shortening of the free surface of the reed. The threshold blowing pressure is proportional to the product of the reed stiffness and the reed tip aperture opening. To first approximation, these changes tend to cancel in this product so that the threshold blowing pressure changes only a little. This tensing of the embouchure decreases the dynamic mass of the reed, also due to the shortening of the free surface of the reed. The increase in stiffness and decrease in dynamic mass both contribute to increasing the reed resonance frequency.

A third factor to consider in modelling the reed as an oscillator is the reed damping, expressed as a quality factor (Q). Worman (1971) measured the dynamical properties of a clarinet reed and found the resonance frequency to be approximately f_r = 2500 Hz and its Q = 3. The corresponding resonance bandwidth of the reed is $\Delta f_r = f_r/Q$ = 800 Hz, which we may as well round up to 1000 Hz to within the accuracy of Worman's measurement. It follows that the frequency range over which the reed resonance effects are important is approximately 2000-3000 Hz, and it is assumed that the reed resonance frequency itself can be placed in this range by the player through changes in embouchure.

For blowing pressures slightly above threshold, Thompson analyzed the role of the reed resonance in small-amplitude acoustic oscillations. It turns out that energy can be injected into the oscillation at the reed resonance frequency if the Q is sufficiently high. The playing frequency is controlled both by the air-column resonances and by the reed resonance. It is this collaboration that qualitatively explains how the player exerts control over the playing frequency by modifications of the reed resonance.

Thompson tested this theory using an artificial blowing machine with an artificial reed. This was a metal reed whose resonance frequency was adjustable in the vicinity of 2500 Hz and whose Q was approximately 10. Thompson observed that variations in the reed resonance frequency modified the playing frequency. Interestingly, he observed entrainment of the playing frequency to a sub-harmonic of the reed resonance frequency, or, alternatively, the playing frequency was set such that some harmonic multiple of it coincided with the reed resonance frequency. The main text assumes that this entrainment phenomenon occurs in actual clarinet notes.

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Yet it is not clear that this entrainment occurs in the clarinet. As yet, no one has devised an experiment to measure the reed resonance frequency while the player sustains a note. In Sec. I, step 3 under the subheading 'To meet a clarinet' suggests that the player can choose two different embouchure settings for B_4 and achieve a 'singing' tone. This suggests that separate regimes of oscillation exist for the same nominal pitch. It is entirely possible for this to be true separate from the issue of whether entrainment exists. For the other playing demonstrations, it appears to be sufficient that an increase or decrease in lip tension produces an increase or decrease in playing frequency. This is much less restrictive than the statement that entrainment occurs.

Using the correspondence of the musical acoustician's regimes of oscillation with the dynamicist's attractors, one can visualize the basin of attraction in the dynamical phase space. One embouchure setting leads to one basin of attraction in which phase space trajectories converge to a limit cycle, that is, a stable, centred, singing tone. Another embouchure setting, with its corresponding perturbation of the reed control parameters, leaves the first basin of attraction and enters another in which trajectories converge to a second limit cycle, that is, an alternative stable, centred, singing tone. Entrainment arises due to the presence of non-linearity. The most common type of entrainment is the entrainment of one non-linear oscillator to the frequency of another, but harmonic and sub-harmonic entrainment are also generic.

Schumacher (1981) demonstrated the feasibility of studying sound production in the clarinet by simulations in the time domain. This approach simplifies the study of even large-amplitude oscillations, that is, if the underlying model can be assumed to be correct. The clarinet model examined by Schumacher was identical to that analyzed in the frequency domain by Worman and Thompson. Since the player's control of the reed resonance of the clarinet can be simulated in the time domain, it is possible to determine whether entrainment occurs in simulated clarinet notes. Entrainment would be observed if there would be a finite range of reed resonance frequencies leading to a playing frequency that is an exact sub-harmonic of the reed frequency.

It turns out that entrainment did not occur in clarinet-like simulations with a reed Q of 3 (Keefe, 1992). These simulations properly accounted for the closing condition of the reed tip with the mouthpiece lay. An increase in the reed resonance frequency did lead to an increase in playing frequency. This accounts for the player's control over intonation through control of embouchure. Since Thompson's experiments used a metal reed with a Q of 10, clarinet-like simulations were also carried out for chalumeau- and clarion-register notes whose reed Qs were in the range of 5 to 15. In no case was entrainment observed, but the playing frequency did increase with the reed resonance frequency, as before.

In related experiments (Keefe and Waeffler, 1993), the reed resonance frequency and the reed Q of clarinet (and saxophone) reeds were measured using impact hammer excitation. The reeds were tested under dry and moist conditions, and the influence of the lower lip on the reed in the actual player's embouchure was simulated by closing the reed with the experimenter's thumb before striking the reed with the hammer. These preliminary results were that the reed resonance frequency varied in the range of 2.8-3.1 kHz, and the

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moist-reed Q, although lower than the dry-reed Q, was measured in the range of 5-10. This is higher than reported by Worman (1971). In addition, the measured response of the struck reed included multiple resonances that are not accounted for in existing models of sound production. This higher range of Q does lead in the existing time-domain simulations to a more pronounced influence of playing frequency by the reed resonance frequency, but the frequencies of the air column resonances also exert control over the playing frequency. Neither set of modes entirely wins out in the normal clarinet notes under investigation, as would be the case if entrainment were found to occur.

There is a final factor to consider in the player's control of the embouchure. Not only are the reed stiffness and dynamic mass (and possibly changes in reed Q) modified by changes in embouchure, but the change in the reed tip aperture modifies the hydrodynamics of the flow through the tip (Hirschberg *et al.*, 1991; Gilbert, 1992). It is this non-linear hydrodynamical relationship that injects the energy needed to sustain the oscillation. Changes in this aperture, for fixed reed parameters and fixed blowing pressure, may significantly affect the hydrodynamics via a change in the flow-separation characteristics within the mouthpiece. Even if the flow-separation characteristics do not vary appreciably over the period in which the reed aperture opens and closes, the player's adjustment of the reed tip aperture via changes in lip tension leads to changes in the non-linear flow-control function that may be more important than the corresponding changes that the lip tension exerts on the mechanical properties of the reed.

The essential difference between this explanation and that in the main text has to do with how the player controls intonation through changes in embouchure. Where the main text uses the language of setting the reed resonance to specific values, this appendix advocates a more general influence of the reed resonance on playing frequency in conjunction with changes in the hydrodynamics at the reed tip, all produced by changes in the player's embouchure. The skilled player makes consistent changes in tuning and shadings in tone colour through adjusting the embouchure.

NOTES

[1] Address correspondence to Keefe at Boys Town National Research Hospital, 555 North 30th Street, Omaha, NE 68131, U.S.A.

[2] Editor's note: The word 'tone' in the original manuscript (in US English) has been changed to 'note' whenever a pitch name is implied, in accordance with British English usage.

[3] Editor's note: We have retained here the authors' occasional use of 'clarion register' (for what is more commonly known in Britain as the 'second-' or 'clarinet register').

[4] This experiment, described in Keefe (1983), has been replicated by Backus (1981) and by Dane, reported in Hirschberg *et al.* (1991), who obtained similar results.

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[5] The criterion of what constitutes 'reasonably large' for the length-to-diameter ratio of a tone hole remains a topic for further research. It is unknown to what extent the differences in playing properties of wind instruments in different historical eras can be related to differences in this tone-hole design parameter; consider, for example, the pronounced undercutting of tone holes on many baroque instruments.

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